Chamley–Judd and Optimal Capital Taxation

Jake Thompson

November 27, 2025

Model Setup

Two agents:

- **Hand-to-mouth workers** who supply one unit of labor inelastically.
- **Idle capitalists** who choose $\{C_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to the capital accumulation constraint:

$$K_{t+1} = (1-\delta)K_t + (1-\tau)r_t K_t - C_t$$

Production and Factor Prices

Cobb–Douglas production and perfect competition imply:

$$r_t K_t = \alpha K_t^{\alpha}$$

So the law of motion becomes:

$$K_{t+1} = (1-\delta)K_t + (1-\tau)\alpha K_t^{\alpha} - C_t$$

Taking partial derivatives:

$$\frac{\partial C_t}{\partial K_{t+1}} = -1, \qquad \frac{\partial C_t}{\partial K_t} = (1-\delta) + (1-\tau)\alpha^2 K_t^{\alpha-1}$$

Dynamic Programming

The value function is:

$$V(K_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \beta V(K_{t+1})$$

First-order condition:

$$\frac{\partial V(K_t)}{\partial K_{t+1}} = -C_t^{-\sigma} + \beta C_{t+1}^{-\sigma}\left((1-\delta) + (1-\tau)\alpha^2 K_{t+1}^{\alpha-1}\right) = 0$$

Euler Equation:

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma}\left((1-\delta) + (1-\tau)\alpha^2 K_{t+1}^{\alpha-1}\right)$$

Steady State Analysis

At steady state, $C_t = C_{t+1} = \bar{C}$ and $K_t = \bar{K}$:

$$\bar{C}^{-\sigma} = \beta \bar{C}^{-\sigma} \left((1-\delta) + (1-\tau)\alpha^2 \bar{K}^{\alpha-1} \right)$$

$$\frac{1}{\beta} = (1-\delta) + (1-\tau)\alpha^2 \bar{K}^{\alpha-1}$$

$$\frac{1}{\beta} - (1-\delta) = (1-\tau)\alpha^2 \bar{K}^{\alpha-1}$$

$$\frac{\beta^{-1} - 1 - \delta}{(1-\tau)\alpha^2} = \bar{K}^{\alpha-1}$$

$$\bar{K} = \left[\frac{(\beta^{-1} - 1 - \delta)}{(1-\tau)\alpha^2} \right]^{\frac{1}{\alpha-1}}$$

Worker's Income

Worker's post-tax income is:

$$W_t = \underbrace{(1-\alpha)K^\alpha}_{\text{wages}} + \underbrace{\tau\alpha K^\alpha}_{\text{tax}}$$

$$W_t = (1 - \alpha + \tau\alpha)K^\alpha$$

At steady state:

$$\bar{W} = (1 - \alpha + \tau\alpha) \left[\frac{(\beta^{-1} - 1 - \delta)}{(1-\tau)\alpha^2} \right]^{\frac{\alpha}{\alpha-1}}$$

Optimal Tax Rate

Maximize $\ln(\bar{W})$:

$$\ln(\bar{W}) = \ln(1 - \alpha + \tau\alpha) + \frac{\alpha}{\alpha - 1} \left[\ln(\beta^{-1} - 1 - \delta) - \ln(1-\tau) - 2\ln(\alpha) \right]$$

First-order condition with respect to τ:

$$\frac{\partial \ln(\bar{W})}{\partial \tau} = \frac{\alpha}{1 - \alpha + \tau\alpha} + \frac{\alpha}{\alpha - 1} \cdot \frac{1}{1-\tau} = 0$$

$$\frac{\alpha}{1-\alpha} \cdot \frac{1}{1-\tau} = \frac{\alpha}{1 - \alpha + \tau\alpha}$$

$$(1-\alpha)(1-\tau) = 1 - \alpha + \tau\alpha$$

$$1 - \tau - \alpha + \alpha\tau = 1 - \alpha + \tau\alpha$$

$$-\tau = 0$$

$$\boxed{\tau = 0}$$